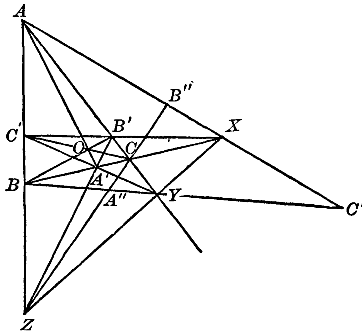
I. Solution by William Hoover, Columbus, Ohio.

Take ABC as the triangle of reference, and the trilinear coördinates of the vertices A, B, C to be $(\alpha_2, 0, 0)$; $(0, \beta_2, 0)$; $(0, 0, \gamma_2)$, and those of O, $(\alpha_1, \beta_1, \gamma_1)$; then the equation of AO is $\gamma_1\beta - \beta_1\gamma = 0$.

The coördinates of A' will be proportional to $(0, \beta_1, \gamma_1)$, and, by symmetry, those of B' to $(\alpha_1, 0, \gamma_1)$, and of C' to $(\alpha_1, \beta_1, 0)$.

The equation of $A'B'$ is $\beta_1\gamma_1\alpha + \alpha_1\gamma_1\beta - \alpha_1\beta_1\gamma = 0$.



The point of intersection of AB and $A'B'$, or Z, has coördinates proportional to $(-\alpha_1, \beta_1, 0)$; those of the intersection BC and $B'C'$, or X, to $(0, -\beta_1, \gamma_1)$; and of CA and $C'A'$, or Y, to $(\alpha_1, 0, -\gamma_1)$; and X, Y, Z are collinear, since

$$\begin{vmatrix} 0, & -\beta_1, & \gamma_1 \\ \alpha_1, & 0, & -\gamma_1 \\ -\alpha_1, & \beta_1, & 0 \end{vmatrix} = \alpha_1\beta_1\gamma_1 \begin{vmatrix} 0, & -1, & 1 \\ 1, & 0, & -1 \\ -1, & 1, & 0 \end{vmatrix}$$

$$= \alpha_1\beta_1\gamma_1 \begin{vmatrix} 0, & 0, & 1 \\ 1, & -1, & -1 \\ -1, & 1, & 0 \end{vmatrix} = \alpha_1\beta_1\gamma_1 \begin{vmatrix} 1, & -1 \\ -1, & 1 \end{vmatrix} = 0.$$

The equation of AX is $\gamma_1\beta + \beta_1\gamma = 0$, and those of BY and CZ respectively $\gamma_1\alpha + \alpha_1\gamma = 0$, and $\beta_1\alpha + \alpha_1\beta = 0$.

The coördinates of C'', the point of intersection of AX and BY, are proportional to $(\alpha_1, \beta_1, \gamma_1)$; of BY and CZ, to $(\alpha_1, \beta_1, \gamma_1)$; and of CZ and AX to $(\alpha_1, \beta_1, \gamma_1)$, the latter two points being A'', B'' respectively.

It is evident now that the equations to AA'', BB'', CC'' are the same as those of AO, BO, CO, in order, the first three lines then passing through O.

II. Solution by H. L. Olson, Chicago, Illinois.

I shall amplify this theorem by proving that the lines AA'', BB'', CC'' are identical with the lines AA', BB', CC' respectively, and hence intersect in the point O. In the triangles $BC'Y$ and $CB'Z$, the lines BC, $C'B'$, and YZ, joining corresponding vertices, meet at X, and hence the points A, A', A'', in which corresponding sides meet, are collinear. Similarly B, B', B'' are collinear; also C, C', C''. Hence the lines AA'', BB'', CC'' meet at O.

Also solved by A. Pelletier and the Proposer.

2716 [June, 1918]. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

To a passenger in a train moving at the rate of 40 miles per hour, the rain appears to be rushing downward and towards him at an angle of 20 degrees with the horizontal. If the rain is actually falling in a vertical direction, show that the velocity of the raindrops in feet per second is 21.35.

Solution by E. H. Worthington, Elkins Park, Pa.

A velocity of 40 miles per hour is the same as $58\frac{2}{3}$ feet per second. If v is the velocity of the raindrop, we have $v = 58\frac{2}{3} \tan 20° = 58\frac{2}{3} \times 0.364$ feet per sec. $= 21.35$ feet per second.

Also solved by H. E. Carleton, A. M. Harding, H. L. Olson, A. Pelletier and J. B. Reynolds.

2735 [December, 1918]. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If two lines AE and BD, drawn from the vertices A and B of a triangle to the opposite sides, divide the angles A and B so that the parts of A are respectively less than the corresponding parts of B, then AE is greater than BD.